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DECAY OF TEMPERATURE FLUCTUATIONS IN HOMOGENEOUS TURBULENCE BEFORE THE FINAL PERIOD*

A. L. LOEFFLER, Jr.† and R. G. DEISSLER
Lewis Research Center,
National Aeronautics and Space Administration, Cleveland, Ohio

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Abstract—A previous analysis of homogeneous turbulence for times before the final period has been extended to the case of temperature fluctuations in a homogeneous turbulence. The method consists essentially of the solution of the 2- and 3-point Fourier-transformed temperature equations after neglecting the fourth-order correlations in comparison with the second- and third-order correlations. Results are obtained for the convective transfer function, the spectral "energy" function and the total temperature fluctuation "energy". Comparison is made between the analysis and published experimental data obtained using air. The decay law obtained may be written $\overline{T^2} = A(t - t_0)^{-3/2} + B(t - t_0)^{-5}$ where $\overline{T^2}$ is the total "energy" (the mean square of the temperature fluctuations), t is the time and A and t_0 are constants determined by the initial conditions. The constant B depends on both initial conditions and the fluid Prandtl number. For large times the last term becomes negligible, leaving the $-3/2$ power decay law for the final period previously found by Corrsin.

It is shown that the effect of increasing Prandtl number is to extend the spectral "energy" function to larger wave numbers and to reduce the rate of decay of the temperature fluctuations.

Résumé—Une étude précédente de la turbulence homogène, à des instants précédant la période finale, a été étendue au cas des fluctuations de température en turbulence homogène. La méthode consiste essentiellement à résoudre les équations aux transformées de Fourier des températures en deux et trois points, après avoir négligé les corrélations du 4° ordre vis à vis des corrélations du 2° et 3° ordre. Des résultats sont obtenus pour les fonctions de la transmission de chaleur, du spectre d'"énergie" et l'"énergie" de fluctuation de la température totale. Une comparaison est effectuée entre la présente étude et les résultats expérimentaux publiés pour l'air. La loi de dégradation obtenue peut s'écrire $\overline{T^2} = A(t - t_0)^{-3/2} + B(t - t_0)^{-5}$ où $\overline{T^2}$ est l'"énergie" totale (moyenne quadratique des fluctuations de température), t le temps A et t_0 sont des constantes déterminées par les conditions initiales. La constante B dépend à la fois des conditions initiales et du nombre de Prandtl. Pour de grandes périodes, le dernier terme devient négligeable, seule subsiste la loi de dégradation en puissance $-3/2$ pour la période finale précédemment trouvée par Corrsin.

On montre que l'effet de l'augmentation du nombre de Prandtl est d'étendre la fonction du spectre d'"énergie" à des nombres d'ondes plus grands et de réduire la vitesse de dégradation des fluctuations de température.

Zusammenfassung—Eine frühere Untersuchung der homogenen Turbulenz für Zeiten vor der Endperiode wurde auf den Fall von Temperaturschwankungen in homogener Turbulenz ausgedehnt. Die Methode besteht im wesentlichen in der Lösung der Fourier-transformierten Temperaturgleichungen für zwei und drei Punkte, unter Vernachlässigung der Korrelationen vierter Ordnung gegen die der zweiten und dritten Ordnung. Man erhält Ergebnisse für die konvektive Übergangsfunktion, die spektrale "Energie"-Funktion und die totale "Energie" der Temperaturschwankungen. Mit Literaturwerten für Luft wird ein Vergleich durchgeführt. Das Abklinggesetz kann in der Form

$$\overline{T^2} = A(t - t_0)^{-3/2} + B(t - t_0)^{-5}$$

geschrieben werden. Hierin bedeuten $\overline{T^2}$ die totale "Energie" (das mittlere Quadrat der Temperatur-

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† Present address: Grumman Aircraft Engineering Corporation, Bethpage, N.Y.

schwankungen), t die Zeit, A und t_0 Konstanten, die aus den Anfangsbedingungen zu bestimmen sind. Die Konstante B hängt sowohl von den Anfangsbedingungen wie von der Prandtlzahl ab. Da für grosse Zeiten der zweite Term zu vernachlässigen ist, bleibt für die Endperiode das Abklinggesetz mit der Potenz-3/2, wie es früher von Corrsin gefunden wurde.

Es zeigt sich ferner, dass bei wachsender Prandtlzahl die spektrale "Energie"-Funktion zu grösseren Wellenzahlen ausgedehnt und das Abklingen der Temperaturschwankungen vermindert wird.

Аннотация—Существующий анализ турбулентности был распространен на случай температурных колебаний в среде с однородной турбулентностью. Метод состоит в решении дифференциальных уравнений переноса тепла с помощью двух-и трёхмерных преобразований Фурье без учёта соотношения преобразований четвертого, второго и третьего порядков. Получены результаты для функций конвективного переноса и спектральной «энергии», а также общей «энергии» температурных колебаний. Сравнивается анализ и опубликованные экспериментальные данные, полученные для воздуха. Закон релаксации может быть записан как

$$\overline{T^2} = A\,(t-t_0)^{-3/2} + B\,(t-t_0)^{-5}$$

где $\overline{T^2}$ — общая «энергия» (среднее квадратичное температурных колебаний), t — время, A и t_0 — постоянные, определяемые начальными условиями. Постоянная B зависит от начальных условий и от критерия Прандтля для жидкости (газа). Для больших промежутков времени последнюю величину не учитываем, оставляя закон релаксации в виде одночлена со степенью —3/2 для конечного периода, открытого ранее Коррсиным.

Показано, что увеличение критерия Прандтля приводит к распространению спектральной функции «энергии» на большие волновые критерии и к уменьшению скорости затухания температурных колебаний.

INTRODUCTION

A THOROUGH study of the decay of temperature fluctuations in homogeneous turbulence would appear to be one of the initial steps required for understanding the important process of heat transfer in shear turbulence. As pointed out in [1], such a study would also be applicable to concentration fluctuations during the mixing of equi-dense fluids, for the cases of constant mutual diffusion coefficient and no interfacial tension.

Corrsin [1, 2] has already made an analytical attack on the problem of turbulent temperature fluctuations using the approaches employed in the statistical theory of turbulence. His results pertain to the final period of decay, and, for the case of appreciable convective effects, to the "energy" spectral form in specific wave-number ranges. Further work along this same line has more recently been done Ogura [7].

In 1958 Deissler [3] presented a theory of homogeneous turbulence which was valid for times before the final period. Essentially, the theory presented in [3] is valid during the period for which the fourth- and higher-order velocity correlation terms are negligible compared to the second- and third-order correlation terms. In [4] the analysis was extended to still earlier times by neglecting only the fifth- and higher-order correlation terms compared to lower-order correlation terms.

In solving the problem of the decay of temperature fluctuations in homogeneous turbulence before the final period it seems logical to use the approach which has already been employed with success for studying turbulence. In this paper, therefore, the method of [3] is used to study decay of temperature fluctuations in homogeneous turbulence. The results of the analysis are compared to the experimental data of [6].

NOTATION

$C(\kappa)$,	function of κ defined by equation (38);
C_p,	heat capacity at constant pressure;
$F(\eta)$,	function defined by equation (40);
G,	spectral "energy" function;
$H^{\oplus}$,	dimensionless quantity defined by equation (52);
K,	spectral transfer function;
k,	thermal conductivity;
M,	mesh size;
N_0,	constant depending on initial conditions;
Pr,	Prandtl number;
p,	static pressure;

R,	dimensionless function of Pr, given by equation (44).
Re_M,	Reynolds number, MU/ν;
$\mathbf{r}$,	displacement vector from P to P';
$\mathbf{r}'$,	displacement vector from P to P'';
$S^{\oplus}$,	dimensionless quantity defined by equation (53);
T,	temperature fluctuation from time average;
$\bar{T}$,	time average value of temperature;
$\tilde{T}$,	instantaneous value of temperature;
t,	time;
U,	time average velocity of fluid in direction perpendicular to grid;
u,	velocity fluctuation from time average;
$\bar{u}$,	time average value of velocity;
$\tilde{u}$,	instantaneous value of velocity;
x,	distance in flow direction from heated grid;
$\mathbf{x}$,	position vector;

Greek symbols

γ,	thermal diffusivity;
δ_0,	constant depending on initial conditions;
ζ,	angle between $\boldsymbol{\kappa}$ and $\boldsymbol{\kappa}'$;
η,	variable defined by equation (41);
$\boldsymbol{\kappa}$, $\boldsymbol{\kappa}'$,	wave-number vectors (magnitude in dimensions of 1/length);
λ_T,	thermal microscale;
ν,	kinematic viscosity;
ρ,	fluid density;
$\tau, \phi, \beta, \alpha, \theta$,	symbols used in quantities obtained by Fourier transforms.

Subscripts

i, j, k, l,	symbol used to indicate direction;
0,	indicates initial conditions;
min,	minimum value;
norm,	normalized quantity.

Superscripts

$'$,	indicates quantities at point P';
$''$,	indicates quantities at point P'';
$\oplus$,	indicates dimensionless quantities.

THEORY

A. Correlation and spectral equations

For an incompressible fluid with constant properties and for negligible frictional heating, the energy equation may be written

$$\rho C_p \left[\frac{\partial \tilde{T}}{\partial t} + \tilde{u}_i \frac{\partial \tilde{T}}{\partial x_i}\right] = k \frac{\partial^2 \tilde{T}}{\partial x_i \partial x_i} \tag{1}$$

where $\tilde{T}$ and $\tilde{u}_i$ are instantaneous values of temperature and velocity. Breaking these instantaneous values into time average and fluctuating components as $\tilde{T} = \bar{T} + T$ and $\tilde{u}_i = \bar{u}_i + u_i$ allows equation (1) to be written

$$\frac{\partial \bar{T}}{\partial t} + \frac{\partial T}{\partial t} + \bar{u}_i \frac{\partial \bar{T}}{\partial x_i} + \bar{u}_i \frac{\partial T}{\partial x_i} + u_i \frac{\partial \bar{T}}{\partial x_i} + u_i \frac{\partial T}{\partial x_i} = \gamma \left[\frac{\partial^2 \bar{T}}{\partial x_i \partial x_i} + \frac{\partial^2 T}{\partial x_i \partial x_i}\right] \tag{2}$$

where $\gamma \equiv k/\rho C_p$. From the condition of homogeneity it follows that $\partial \bar{T}/\partial x_i = 0$, and in addition the usual assumption is made that $\bar{T}$ is independent of time and that $\bar{u}_i = 0$. Thus equation (2) simplifies to

$$\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = \left(\frac{\nu}{Pr}\right) \frac{\partial^2 T}{\partial x_i \partial x_i} \tag{3}$$

where $Pr = \nu/\gamma$.

Equation (3) is assumed to hold at the arbitrary point P. For point P' the corresponding equation can be written

$$\frac{\partial T'}{\partial t} + u_i' \frac{\partial T'}{\partial x_i'} = \left(\frac{\nu}{Pr}\right) \frac{\partial^2 T'}{\partial x_i' \partial x_i'} \tag{4}$$

Multiplying equation (3) by T', equation (4) by T, time averaging and adding the two equations gives

$$\frac{\partial \overline{TT'}}{\partial t} + \overline{u \frac{\partial (TT')}{\partial x_i}} + \overline{u_i' \frac{\partial (TT')}{\partial x_i'}} = \frac{\nu}{Pr}\left[\frac{\partial^2 (\overline{TT'})}{\partial x_i \partial x_i} + \frac{\partial^2 (\overline{TT'})}{\partial x_i' \partial x_i'}\right] \tag{5}$$

The continuity equation is

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial u_i'}{\partial x_i'} = 0 \tag{6}$$

Substitution of equation (6) into (5) yields

$$\frac{\partial(\overline{TT'})}{\partial t}+\frac{\partial(\overline{u_iTT'})}{\partial x_i}+\frac{\partial(\overline{u_i'TT'})}{\partial x_i'}=\frac{\nu}{Pr}\left[\frac{\partial^2(\overline{TT'})}{x_i'\partial x_i}+\frac{\partial^2(\overline{TT'})}{\partial x_i'\partial x_i'}\right] \quad (7)$$

By use of a new independent variable,

$$r_i \equiv x_i' - x_i,$$

it is possible to rewrite equation (7) in the form

$$\frac{\partial(\overline{TT'})}{\partial t}-\frac{\partial(\overline{u_iTT'})}{\partial r_i}+\frac{\partial(\overline{u_i'TT'})}{\partial r_i}=\frac{2\nu}{Pr}\frac{\partial^2(\overline{TT'})}{\partial r_i\partial r_i} \quad (8)$$

It is convenient to write this equation in spectral form by use of the following three-dimensional Fourier transforms

$$\overline{TT'}(\mathbf{r})=\int_{-\infty}^{\infty}\overline{\tau\tau'}(\boldsymbol{\varkappa})\exp(i\boldsymbol{\varkappa}\cdot\mathbf{r})\,d\boldsymbol{\varkappa} \quad (9)$$

$$\overline{u_i'TT'}(\mathbf{r})=\int_{-\infty}^{\infty}\overline{\phi_i\tau\tau'}(\boldsymbol{\varkappa})\exp(i\boldsymbol{\varkappa}\cdot\mathbf{r})\,d\boldsymbol{\varkappa} \quad (10)$$

and, since it is obvious by interchanging P and P' that $\overline{u_i'TT'}(\mathbf{r})=\overline{u_iTT'}(-\mathbf{r})$,

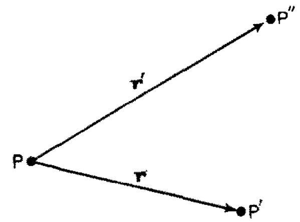
$$\overline{u_iTT'}(\mathbf{r})=\int_{-\infty}^{\infty}\overline{\phi_i\tau\tau'}(-\boldsymbol{\varkappa})\exp(i\boldsymbol{\varkappa}\cdot\mathbf{r})\,d\boldsymbol{\varkappa} \quad (11)$$

Substitution of equations (9)–(11) into equation (8) leads to the spectral equation

$$\frac{\partial\overline{\tau\tau'}(\boldsymbol{\varkappa})}{\partial t}+i\kappa_i\left[\overline{\phi_i\tau\tau'}(-\boldsymbol{\varkappa})-\overline{\phi_i\tau\tau'}(\boldsymbol{\varkappa})\right]=-\frac{2\nu}{Pr}\kappa^2\overline{\tau\tau'}(\boldsymbol{\varkappa}) \quad (12)$$

Equation (12) is analogous to the two-point spectral equation governing the decay of velocity fluctuations (as pointed out in [1]) and therefore the quantity $\overline{\tau\tau'}(\boldsymbol{\varkappa})$ may be interpreted as a temperature fluctuation "energy" contribution of thermal eddies of size $1/\kappa$. Equation (12) expresses the time derivative of this "energy" as a function of the convective transfer to other wave numbers and the "dissipation" due to the action of thermal conductivity. The second term on the left-hand side of equation (12) is the so-called transfer term while the term on the right-hand side is the "dissipation" term.

In order to obtain an additional relation involving the unknown quantities the same general procedure is followed as in [3] and a three-point equation is derived. For this purpose the three points P, P' and P'' together with the indicated position vectors $\mathbf{r}$ and $\mathbf{r}'$ are considered.



For the two points P' and P'' a relation analogous to equation (7) can be found as

$$\frac{(\partial T'T'')}{\partial t}+\frac{\partial(u_i'T'T'')}{\partial x_i'}+\frac{\partial(u_i''T'T'')}{\partial x_i''}=\frac{\nu}{Pr}\left\{\frac{\partial^2(T'T'')}{\partial x_i'\partial x_i'}+\frac{\partial^2(T'T'')}{\partial x_i''\partial x_i''}\right\} \quad (13)$$

If equation (13) is multiplied through by u_j, the jth velocity fluctuation component at point P, the resulting equation can be written in the form

$$\frac{\partial(u_jT'T'')}{\partial t}+\frac{\partial(u_ju_i'T'T'')}{\partial x_i'}+\frac{\partial(u_ju_i''T'T'')}{\partial x_i''}=\frac{\nu}{Pr}\left\{\frac{\partial^2(u_jT'T'')}{\partial x_i'\partial x_i'}+\frac{\partial^2(u_jT'T'')}{\partial x_i''\partial x_i''}\right\}+T'T''\frac{\partial u_j}{\partial t} \quad (14)$$

The momentum equation at point P is

$$\frac{\partial u_j}{\partial t}+\frac{\partial(u_ju_i)}{\partial x_i}=-\frac{1}{\rho}\frac{\partial p}{\partial x_j}+\nu\frac{\partial^2u_j}{\partial x_i\partial x_i} \quad (15)$$

If equation (15) is solved for $\partial u_j/\partial t$ and substituted into equation (14), the result on taking time averages is

$$\left.\begin{aligned}&\frac{\partial(\overline{u_jT'T''})}{\partial t}+\frac{\partial(\overline{u_ju_i'T'T''})}{\partial x_i}+\frac{\partial(\overline{u_ju_i''T''T'})}{\partial x_i''}=\\&\quad\frac{\nu}{Pr}\left\{\frac{\partial^2(\overline{u_jT'T''})}{\partial x_i'\partial x_i'}+\frac{\partial^2(\overline{u_jT'T''})}{\partial x_i''\partial x_i''}\right\}-\\&\quad-\frac{\partial(\overline{u_ju_iT'T''})}{\partial x_i}-\frac{1}{\rho}\frac{\partial(\overline{pT'T''})}{\partial x_j}+\\&\quad+\nu\frac{\partial^2(\overline{u_jT'T''})}{\partial x_i\partial x_i}\end{aligned}\right\} \quad (16)$$

Making use of the relations $r_i = x'_i - x_i$ and $r'_i = x''_i - x_i$ allows equation (16) to be rewritten as

$$\left.\begin{aligned} &\frac{\partial(\overline{u_jT'T''})}{\partial t} - \frac{\nu}{Pr}\left\{(1+Pr)\frac{\partial^2(\overline{u_jT'T''})}{\partial r_i \partial r_i} + \right.\\ &\left. + 2Pr\frac{\partial^2(\overline{u_jT'T''})}{\partial r_i \partial r'_i} + (1+Pr)\frac{\partial^2(\overline{u_jT'T''})}{\partial r'_i \partial r'_i}\right\}\\ &= -\frac{\partial(\overline{u_ju'_iT'T''})}{\partial r_i} - \frac{\partial(\overline{u_ju''_iT''T'})}{\partial r'_i} + \\ &+ \frac{\partial(\overline{u_ju_iT'T''})}{\partial r'_i} + \frac{\partial(\overline{u_ju_iT'T''})}{\partial r_i} + \\ &+ \frac{1}{\rho}\frac{\partial(\overline{pT'T''})}{\partial r'_j} + \frac{1}{\rho}\frac{\partial(\overline{pT'T''})}{\partial r_j} \end{aligned}\right\} \quad (17)$$

Six-dimensional Fourier transforms for quantities in this equation may be defined as

$$\overline{u_jT'T''} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\beta_j\theta'\theta''} \exp\left[i(\varkappa\cdot\mathbf{r} + \varkappa'\cdot\mathbf{r}')\right] d\varkappa\, d\varkappa' \quad (18)$$

$$\overline{u_ju'_iT'T''} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\beta_j\beta'_i\theta'\theta''} \exp\left[i(\varkappa\cdot\mathbf{r} + \varkappa'\cdot\mathbf{r}')\right] d\varkappa\, d\varkappa' \quad (19)$$

$$\overline{pT'T''} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{\alpha\theta'\theta''} \exp\left[i(\varkappa\cdot\mathbf{r} + \varkappa'\cdot\mathbf{r}')\right] d\varkappa\, d\varkappa' \quad (20)$$

Interchanging P' and P'' shows that

$$\overline{u_ju''_iT''T'} = \overline{u_ju'_iT'T''}.$$

By use of this fact and equations (18)–(20), equation (17) may be transformed to

$$\frac{\partial(\overline{\beta_j\theta'\theta''})}{\partial t} + \frac{\nu}{Pr}\{(1+Pr)\kappa^2 + 2Pr\,\kappa_i\kappa'_i + + (1+Pr)\kappa'^2\}\,\overline{\beta_j\theta'\theta''} = -\,i(\kappa_i + \kappa'_i)\overline{\beta_j\beta'_i\theta'\theta''} + + i(\kappa'_i + \kappa_i)\overline{\beta_j\beta_i\theta'\theta''} + \frac{1}{\rho}\,i(\kappa_j + \kappa'_j)\overline{\alpha\theta'\theta''} \quad (21)$$

If the derivative with respect to x_l is taken of the momentum equation for point P, the equation multiplied through by $T'T''$ and time averages taken, the resulting equation is

$$\frac{\partial^2(\overline{u_lu_kT'T''})}{\partial x_l \partial x_k} = -\frac{1}{\rho}\frac{\partial^2(\overline{pT'T''})}{\partial x_l \partial x_l} \quad (22)$$

or, in terms of the displacement vectors $\mathbf{r}$ and $\mathbf{r}'$ this becomes

$$\left[\frac{\partial^2}{\partial r'_l \partial r'_k} + 2\frac{\partial^2}{\partial r'_l \partial r_k} + \frac{\partial^2}{\partial r_l \partial r_k}\right](\overline{u_lu_kT'T''}) = -\frac{1}{\rho}\left[\frac{\partial^2}{\partial r'_l \partial r'_l} + 2\frac{\partial^2}{\partial r'_l \partial r_l} + \frac{\partial^2}{\partial r_l \partial r_l}\right](\overline{pT'T''}) \quad (23)$$

Taking the Fourier transform of equation (23) and solving the transformed equation for $\overline{\alpha\theta'\theta''}$ yields

$$\overline{\alpha\theta'\theta''} = \frac{-\rho[\kappa'_l\kappa'_k + 2\kappa'_l\kappa_k + \kappa_l\kappa_k]}{[\kappa'_l\kappa'_l + 2\kappa'_l\kappa_l + \kappa_l\kappa_l]}\overline{\beta_l\beta_k\theta'\theta''} \quad (24)$$

Equation (24) can be used to eliminate $\overline{\alpha\theta'\theta''}$ from equation (21).

B. *Solution for times before the final period*

To obtain the equation for final period decay the third-order fluctuation terms are neglected compared to the second-order terms. Analogously, it would be anticipated that for times before but sufficiently near to the final period the fourth-order fluctuation terms should be negligible in comparison with the third-order terms. If this assumption is made then equation (24) shows that the term $\overline{\alpha\theta'\theta''}$, associated with the pressure fluctuations, should also be neglected. Thus equation (21) simplifies to

$$\frac{\partial\overline{\beta_j\theta'\theta''}}{\partial t} + \frac{\nu}{Pr}\{(1+Pr)\kappa^2 + 2Pr\,\kappa_i\kappa'_i + + (1+Pr)\kappa'^2\}\,\overline{\beta_j\theta'\theta''} = 0 \quad (25)$$

Inner multiplication of equation (25) by κ_j and integration between t_0 and t gives

$$\kappa_j\overline{\beta_j\theta'\theta''} = [\kappa_j\overline{\beta_j\theta'\theta''}]_0 \exp\left\{-\frac{\nu}{Pr}[(1+Pr)\kappa^2 + + 2Pr\kappa\kappa'\cos\zeta + (1+Pr)\kappa'^2](t-t_0)\right\} \quad (26)$$

Letting $\mathbf{r}' = 0$ in equation (18) and comparing the result with equation (10) shows that

$$\kappa_i\overline{\phi_{iTT'}}(\varkappa) = \int_{-\infty}^{\infty} \kappa_i\overline{\beta_i\theta'\theta''}\, d\varkappa' \quad (27)$$

Substitution of equations (26) and (27) into equation (12) results in

$$\frac{\partial}{\partial t}\overline{\tau\tau'}(\varkappa) + \frac{2\nu}{Pr}\kappa^2\overline{\tau\tau'}(\varkappa) = \int_{-\infty}^{\infty} i\,[\kappa_i\overline{\beta_i\theta'\theta''} - \kappa_i\overline{\beta_i\theta'(-\varkappa)\theta''(-\varkappa')}]_0 \exp\left\{-\frac{\nu(t-t_0)}{Pr}\left[(1+Pr)\kappa^2 + 2Pr\kappa\kappa'\cos\zeta + (1+Pr)\kappa'^2\right]\right\}d\varkappa' \quad (28)$$

Now $d\varkappa'(\equiv d\kappa_1'd\kappa_2'd\kappa_3')$ can be expressed in terms of κ' and ζ as

$$d\varkappa' = -2\pi\kappa'^2\,d(\cos\zeta)d\kappa' \quad (29)$$

Substitution of equation (29) in equation (28) yields

$$\frac{\partial}{\partial t}\overline{\tau\tau'}(\varkappa) + \frac{2\nu}{Pr}\kappa^2\overline{\tau\tau'}(\varkappa) = \int_0^{\infty} 2\pi i\kappa_i\,[\overline{\beta_i\theta'\theta''} - \overline{\beta_i\theta'(-\varkappa)\theta''(-\varkappa')}]_0\varkappa'^2 \left[\int_{-1}^{1}\exp\left\{\frac{-\nu(t-t_0)}{Pr}[(1+Pr)\kappa^2 + 2Pr\kappa\kappa'\cos\zeta + (1+Pr)\kappa'^2]\right\}d(\cos\zeta)\right]d\kappa' \quad (30)$$

In order to perform the indicated integrations in equations (30) the initial condition,

$$i\kappa_i\,[\overline{\beta_i\theta'\theta''} - \overline{\beta_i\theta'(-\varkappa)\theta''(-\varkappa')}]_0,$$

must be specified. A function which would appear to satisfy all the necessary conditions, as will be seen later, is

$$i\kappa_i\,[\overline{\beta_i\theta'\theta''} - \overline{\beta_i\theta'(-\varkappa)\theta''(-\varkappa')}]_0 = -\frac{\delta_0}{(2\pi)^2}(\kappa^2\kappa'^4 - \kappa^4\kappa'^2) \quad (31)$$

where δ_0 is a constant depending on the initial conditions. Substitution of equation (31) into (30), carrying out the two integrations and defining

$$G(\kappa) = 2\pi\kappa^2\overline{\tau\tau'}(\varkappa) \quad (32)$$

the resulting equation is

$$\frac{\partial G}{\partial t} + \frac{2\nu\kappa^2}{Pr}G = K \quad (33)$$

where

$$K = -\frac{\delta_0}{2\nu(t-t_0)}\int_0^{\infty}(\kappa^3\kappa'^5 - \kappa^5\kappa'^3)\left[\exp\left\{-\frac{\nu(t-t_0)}{Pr}[(1+Pr)\kappa^2 - 2Pr\kappa\kappa' + (1+Pr)\kappa'^2]\right\} - \exp\left\{\frac{-\nu(t-t_0)}{Pr}[(1+Pr)\kappa^2 + 2Pr\kappa\kappa' + (1+Pr)\kappa'^2]\right\}\right]d\kappa' \quad (33a)$$

or, on carrying out the integration,

$$\boxed{K = \frac{-\sqrt{(\pi)}\,\delta_0(Pr)^{5/2}\exp[-\nu(t-t_0)(1+2Pr)\kappa^2/Pr(1+Pr)]}{2\nu^{3/2}(t-t_0)^{3/2}(1+Pr)^{5/2}} \left\{\frac{15Pr\,\kappa^4}{4\nu^2(t-t_0)^2(1+Pr)} + \left[\frac{5Pr^2}{(1+Pr)^2} - \frac{3}{2}\right]\frac{\kappa^6}{\nu(t-t_0)} + \left[\frac{Pr^3}{(1+Pr)^3} - \frac{Pr}{(1+Pr)}\right]\kappa^8\right\}} \quad (34)$$

(It may be of interest to note that the spectral energy function defined in [1] is larger than that defined in equation (32) by a factor of 4.) Since for low κ values, as shown in [1], a Taylor's series expansion for G begins with the product of a time independent constant and κ^2, equation (33) indicates that K must begin as κ^4 for small κ. This condition of K is fulfilled by equation (34).

It can be shown, using equation (34) that

$$\int_0^{\infty} K\,d\kappa = 0 \quad (35)$$

This was to be expected physically since K is a measure of the transfer of "energy" and the total "energy" transferred to all wave numbers must be zero. The necessity for equation (35) to hold can be shown rigorously as follows: if equation (10) is written for both $\varkappa$ and $-\varkappa$, and the resulting equations differentiated with respect to r_i and added, the result is, for

$$\mathbf{r} = 0\left(\frac{\partial}{\partial r_i} = -\frac{\partial}{\partial x_i}\right)$$

$$-2\frac{\partial}{\partial x_i}\overline{u_iTT} = \int_{-\infty}^{\infty} i\kappa_i\,[\overline{\phi_i\tau\tau'}(\varkappa) - \overline{\phi_i\tau\tau'}(-\kappa)]d\varkappa$$

Since according to equations (32), (33) and (12)

$$K \equiv 2\pi i \kappa^2 \kappa_i \,[\overline{\phi_{iTT'}}\,(-\boldsymbol{\kappa}) - \overline{\phi_{iTT'}}\,(\boldsymbol{\kappa})]$$

the previous equation can be written

$$-2\frac{\partial}{\partial x_i}\overline{u_i TT} = \int_{-\infty}^{\infty} \frac{K}{2\pi\kappa^2} \mathrm{d}\boldsymbol{\kappa}$$

Inasmuch as $\mathrm{d}\boldsymbol{\kappa} = 4\pi\kappa^2 \mathrm{d}\kappa$ for $K = K(\kappa, t)$, the last equation becomes

$$\int_0^{\infty} K\,\mathrm{d}\kappa = -\frac{\partial}{\partial x_i}\overline{u_i TT} = 0$$

by homogeneity, Q.E.D.

Since equation (33) is a linear differential equation it can be solved for G as

$$G = \exp\left[-\frac{2\nu\kappa^2(t-t_0)}{Pr}\right]\int K \exp\left[\frac{2\nu\kappa^2(t-t_0)}{Pr}\right] dt + C(\kappa)\exp\left[\frac{-2\nu\kappa^2(t-t_0)}{Pr}\right] \quad (36)$$

where $C(\kappa)$ is an arbitrary function of κ. For large times, Corrsin [1] has shown the correct form of the expression for G to be

$$G = \frac{N_0}{\pi}\kappa^2 \exp\left[\frac{-2\nu\kappa^2(t-t_0)}{Pr}\right] \quad (37)$$

where N_0 is a constant, analogous to the Loitsianskii invariant, which depends on the initial conditions. Using equation (37) to evaluate $C(\kappa)$ in equation (36) yields

$$C(\kappa) = \frac{N_0\kappa^2}{\pi} \quad (38)$$

Substitution of the values of K and $C(\kappa)$ as given by equations (34) and (38) into equation (36) gives the equation

$$\left.\begin{aligned} G(\kappa, t) = {} & \frac{N_0}{\pi}\kappa^2 \exp\left[\frac{-2\nu\kappa^2(t-t_0)}{Pr}\right] + \\ & + \frac{\sqrt{(\pi)}\,\delta_0 (Pr)^{5/2} \exp\,[-\nu\kappa(t-t_0)(1+2Pr)/Pr\,(1+Pr)]}{2\nu^{3/2}(1+Pr)^{7/2}} \\ & \left\{\frac{3Pr\,\kappa^4}{2\nu^2(t-t_0)^{5/2}} + \frac{Pr(7Pr-6)\kappa^6}{3\nu(1+Pr)(t-t_0)^{3/2}} - \frac{4(3Pr^2-2Pr+3)\kappa^8}{3(1+Pr)^2(t-t_0)^{1/2}} + \right. \\ & \left. + \frac{8\sqrt{(\nu)}\,(3Pr^2-2Pr+3)\kappa^9 F(\eta)}{3(1+Pr)^{5/2}\sqrt{(Pr)}}\right\} \end{aligned}\right\} \quad (39)$$

where

$$F(\eta) = \exp(-\eta^2)\int_0^{\eta} \exp(S^2)\,\mathrm{d}S \quad (40)$$

$$\eta = \kappa\sqrt{\left[\frac{\nu(t-t_0)}{Pr(1+Pr)}\right]} \quad (41)$$

The function $F(\eta)$ has been calculated numerically and tabulated in [5]. If in equation (9) $\mathbf{r}$ is set equal to zero and use is made of the definition of G as given by equation (32), the result is

$$\frac{\overline{T^2}}{2} = \int_0^{\infty} G(\kappa)\mathrm{d}\kappa \quad (42)$$

Substituting equation (39) into (42) gives

$$\frac{\overline{T^2}}{2} = \frac{N_0(Pr)^{3/2}}{8\sqrt{(2\pi)}\nu^{3/2}(t-t_0)^{3/2}} + \frac{\delta_0 R}{\nu^6(t-t_0)^5} \quad (43)$$

where R is a function of Prandtl number,

$$\left.\begin{aligned} R = {} & \frac{\pi(Pr)^6}{2(1+Pr)(1+2Pr)^{5/2}} \\ & \left\{\frac{9}{16} + \frac{5Pr(7Pr-6)}{16(1+2Pr)} - \frac{35Pr(3Pr^2-2Pr+3)}{8(1+2Pr)^2} + \right. \\ & + \frac{1{\cdot}5422Pr(3Pr^2-2Pr+3)(1+2Pr)^{5/2}}{\sqrt{(\pi)}\,(1+Pr)^{11/2}} \\ & \left.\left[1 + \sum_{n=1}^{\infty}\frac{(11)\ldots[11+2(n-1)]}{(2n+1)n!(2)^{2n}(1+Pr)^n}\right]\right\} \end{aligned}\right\} \quad (44)$$

The second term on the right-hand side of equation (43) becomes negligible at large times leaving the final period decay law previously found by Corrsin [1].

If use is made of the relation $t = \bar{x}/U$ where $\bar{x}$ is the distance from the heated grid and U the mean fluid velocity, equation (43) can be put in the alternative form

$$\frac{\overline{T^2}}{2} = \frac{N_0(Pr)^{3/2}(U/M)^{3/2}}{8\sqrt{(2\pi)}\nu^{3/2}(\bar{x}/M - \bar{x}_0/M)^{3/2}} + \frac{\delta_0(U/M)^5 R}{\nu^6(\bar{x}/M - \bar{x}_0/M)^5} \tag{45}$$

where M is the grid mesh size.

It is convenient to have the equations in dimensionless form. For this purpose the following dimensionless quantities are defined:

$$\kappa^{\oplus} = \sqrt{[\nu(t-t_0)]}\kappa \quad K^{\oplus} = \frac{\delta_0^{4/7} K}{\nu^{11/7} N_0^{11/7}} \quad t^{\oplus} = \frac{N_0^{2/7}\nu^{9/7}(t-t_0)}{\delta_0^{2/7}} \quad G^{\oplus} = \frac{\nu(t-t_0)G}{N_0}$$

$$\overline{T^2}^{\oplus} = \frac{\delta_0^{3/7}\overline{T^2}}{N_0^{10/7}\nu^{3/7}} \tag{46}$$

By making use of relations (46), equations (34), (39) and (43) can be written in dimensionless form as

$$K^{\oplus} = \frac{-\sqrt{(\pi)}(Pr)^{7/2}\exp\left[-\kappa^{\oplus 2}(1+2Pr)/Pr(1+Pr)\right]}{2(1+Pr)^{5/2}t^{\oplus 11/2}}$$
$$\left\{\frac{15\kappa^{\oplus 4}}{4(1+Pr)} + \left[\frac{5Pr}{(1+Pr)^2} - \frac{3}{2Pr}\right]\kappa^{\oplus 6} + \left[\frac{Pr^2}{(1+Pr)^3} - \frac{1}{(1+Pr)}\right]\kappa^{\oplus 8}\right\} \tag{47}$$

and

$$\left.\begin{aligned} G^{\oplus} &= \frac{\kappa^{\oplus 2}\exp(-2\kappa^{\oplus 2}/Pr)}{\pi} + \frac{\sqrt{(\pi)}Pr^{7/2}\exp\left[-\kappa^{\oplus 2}(1+2Pr)/Pr(1+Pr)\right]}{2(1+Pr)^{7/2}t^{\oplus 7/2}} \\ &\left\{\frac{3\kappa^{\oplus 4}}{2} + \frac{1}{3}\left[\frac{7Pr-6}{1+Pr}\right]\kappa^{\oplus 6} - \frac{4(3Pr^2-2Pr+3)\kappa^{\oplus 8}}{3Pr(1+Pr)^2} + \frac{8(3Pr^2-2Pr+3)\kappa^{\oplus 9}}{3Pr^{3/2}(1+Pr)^{5/2}} \right. \\ &\left. F\left[\sqrt{\left(\frac{\kappa^{\oplus 2}}{Pr(1+Pr)}\right)}\right]\right\} \end{aligned}\right\} \tag{48}$$

and

$$\frac{\overline{T^2}^{\oplus}}{2} = \frac{Pr^{3/2}}{8\sqrt{(2\pi)}\,t^{\oplus 3/2}} + \frac{R}{t^{\oplus 5}} \tag{49}$$

Another quantity of interest is the thermal microscale λ_T, which, for the isotropic case, can be defined by the equation

$$\frac{d\overline{T^2}}{dt} = \frac{-12\gamma\overline{T^2}}{\lambda_T^2} \tag{50}$$

Combination of equations (45) and (50) yields

$$\left(\frac{\lambda_T}{M}\right)^2 = \frac{\dfrac{3(-\bar{x}_0/M)^{3/2}}{\sqrt{(2\pi)}Re_M Pr[(\bar{x}/M)-(\bar{x}_0/M)^{3/2}]} + \dfrac{24R(Pr)\delta_0^{\oplus}(-\bar{x}_0/M)^5}{(Pr)^{5/2}Re_M[(\bar{x}/M)-(\bar{x}_0/M)]^5}}{\dfrac{3(-\bar{x}_0/M)^{3/2}}{8\sqrt{(2\pi)}[(\bar{x}/M)-(\bar{x}_0/M)]^{5/2}} + \dfrac{10R\delta_0^{\oplus}(-\bar{x}_0/M)^5}{Pr^{3/2}[(\bar{x}/M)-(\bar{x}_0/M)]^6}} \tag{51}$$

where Re_M is defined as MU/ν. For large times of decay the second terms in both numerator and denominator become negligibly small and the final period expression for $(\lambda_T/M)^2$ previously derived by Corrsin [2] is obtained.

DISCUSSION AND RESULTS

The experimental data of [6] were obtained behind a heated grid in a wind tunnel using air as fluid. Both temperature and velocity fluctuations were measured with hot wire anemometers. Fig. 1 shows the temperature fluctuation data plotted as $[1/\overline{T^2}]^{2/3}$ against $\bar{x}/M$. The solid line represents the theory as given by equation (45), with the constants $\bar{x}_0/M$, N_0, and δ_0 evaluated as shown. The dotted straight line is the final period curve, as indicated by the first term on the right-hand side of equation (45). The data scatter considerably—it would appear that with present measuring techniques there is less precision in temperature fluctuation measurements than in velocity fluctuation measurements. One of the main reasons for this is that the temperature fluctuations must be kept small to avoid fluid property variations. Comparison of the solid curve with the data indicates that the theory is valid for all $\bar{x}/M$ greater than about 25. None of the data is in the final period.

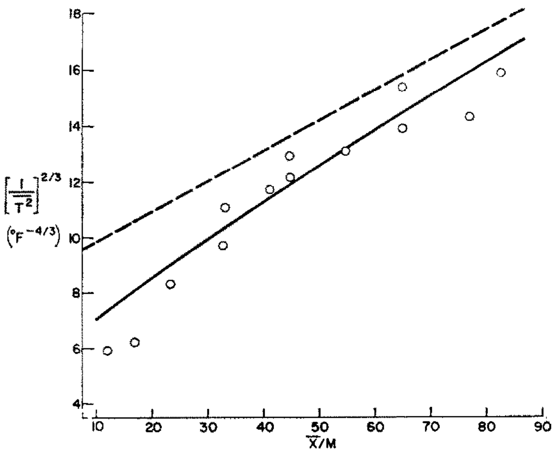


FIG. 1. Theoretical decay of temperature fluctuations compared with experimental data for $Pr = 0{\cdot}72$.
—— Theory, equation (45).
$\bar{X}_0/M = -82{\cdot}6$; $N_0 = 5{\cdot}06 \times 10^{-7}$;
$\delta_0 = 5{\cdot}34 \times 10^{-25}$.
— — Theoretical final period.
○ Data of [6].

The effect of the convective terms can be seen in Fig. 2, where the transfer function $K^{\oplus}$ is plotted against $\kappa^{\oplus}$ for various dimensionless times $t^{\oplus}$ for a Prandtl number of 0·72. Since $K^{\oplus}$ is proportional to the net transfer of thermal "energy" into an eddy size, Fig. 2 shows that there is an overall transfer of "energy" from large eddies to small eddies. The integrated transfer term $\int_0^\infty K^{\oplus} d\kappa^{\oplus}$, must, however, be zero. The curves indicate that, as expected, the transfer term dies out with increasing time. The results are quite analogous to those found by Deissler [3] for the velocity transfer function.

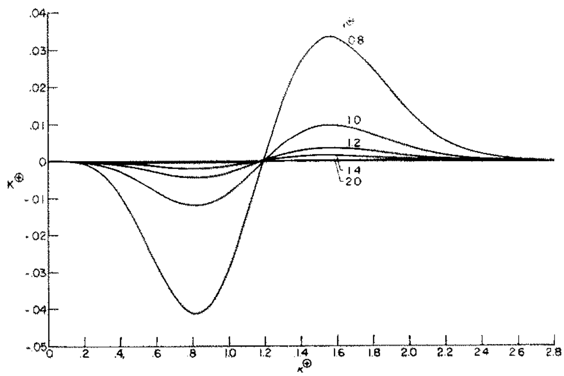


FIG. 2. Dimensionless transfer function $K^{\oplus}$ as a function of wave number and time for $Pr = 0{\cdot}72$.

If a quantity H is defined as the integrand in equation (33a) multiplied by the coefficient before the integral, then a dimensionless quantity $H^{\oplus}$ can be defined by the equation

$$H^{\oplus} \equiv \frac{\nu^5(t-t_0)^5}{\delta_0} H = -\frac{\kappa^{\oplus 8}}{2}\left[\left(\frac{\kappa'}{\kappa}\right)^5 - \left(\frac{\kappa'}{\kappa}\right)^3\right] \left\{\exp\left[-\frac{\kappa^{\oplus 2}}{Pr}\left((1+Pr) - 2Pr\frac{\kappa'}{\kappa} + (1+Pr)\left(\frac{\kappa'}{\kappa}\right)^2\right)\right] - \exp\left[-\frac{\kappa^{\oplus 2}}{Pr}\left((1+Pr) + 2Pr\frac{\kappa'}{\kappa} + (1+Pr)\left(\frac{\kappa'}{\kappa}\right)^2\right)\right]\right\} \quad (52)$$

$H^{\oplus}$ is proportional to the amount of thermal "energy" transferred to a wave number κ from the wave number κ'. In Fig. 3 $H^{\oplus}$ is plotted against κ'/κ for three different values of $\kappa^{\oplus}$ and for $Pr = 0{\cdot}72$. The three values of $\kappa^{\oplus}$ chosen were such that $K^{\oplus}$ was a minimum, zero and a maximum. As an example of the physical significance of the curves, the curve for $\kappa^{\oplus} = 0{\cdot}814$ represents a minimum for $K^{\oplus}$, so that there is an overall loss by convective transfer. The curve demonstrates this overall loss, since it can be seen that more energy is lost by wave number κ to wave numbers greater than κ than is gained from wave numbers less than κ.

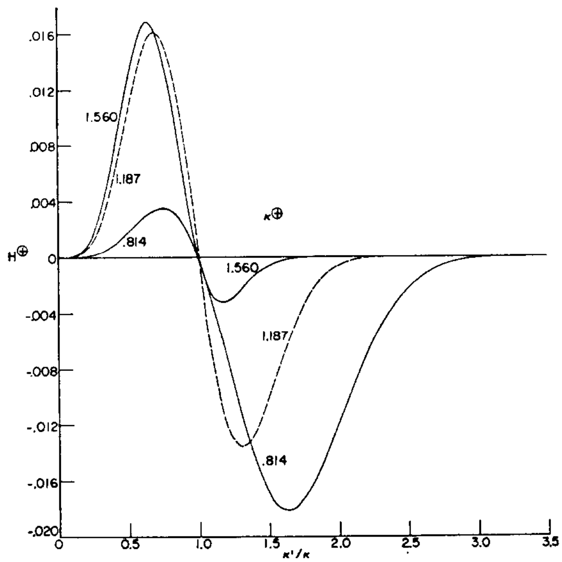


FIG. 3. Variation of dimensionless transfer quantity $H^{\oplus}$ with κ'/κ for three values of $\kappa^{\oplus}$ and $Pr = 0{\cdot}72$.

Fig. 4 is somewhat similar to Fig. 3 except that the effect of κ on the curves has been averaged out by performing an integral of $H^{\oplus}$ from 0 to ∞, obtaining $S^{\oplus}$, or

$$S^{\oplus} = \int_0^{\infty} \{H^{\oplus}\}_{\kappa'/\kappa = \text{const.}} \, d\kappa^{\oplus} \tag{53}$$

$S^{\oplus}$ may be interpreted as being proportional to $H^{\oplus}$ for an average value of κ. Thus, on the average, there is a maximum transfer of "energy" to κ from a wave number κ' equal to about $0{\cdot}65\kappa$, while the maximum loss from κ is to a wave number κ' equal to about $1{\cdot}4\kappa$.

The quantities $H^{\oplus}$ and $S^{\oplus}$ are analogous to quantities defined by Deissler [3] in connexion with the velocity inertial transfer, and the trends found here are quite similar to those.

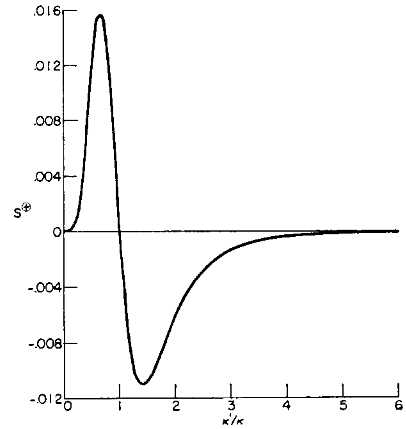


FIG. 4. Variation of averaged transfer quantity $S^{\oplus}$ with κ'/κ for $Pr = 0{\cdot}72$.

In Figs. 5(a), (b) and (c) the transfer function is plotted in the form of $K^{\oplus}t^{\oplus 11/2}$, as found from equation (47), against the dimensionless wave number $\kappa^{\oplus}$ for Prandtl numbers of $0{\cdot}1$, $0{\cdot}72$ and $10{\cdot}0$, respectively. One obvious effect of increasing Prandtl number is to extend the curves out to higher wave numbers. This is to be expected since for higher Prandtl number the thermal conductivity is less and hence there is less tendency for the small thermal eddies to be smeared out.

That changing Prandtl number does not greatly change the shape of the $K^{\oplus}$ curves is shown in Fig. 6, where the ordinates of Fig. 5 have been normalized by division by the absolute value of $[K^{\oplus}t^{\oplus 11/2}]_{\min}$ and the abscissas normalized by division by the value of $\kappa^{\oplus}$ yielding zero ordinate. Thus it would appear that the basic nature of the convective transfer is unaltered by a change in the Prandtl number.

In Figs. 7(a), (b) and (c) curves are presented for the dimensionless spectral function $G^{\oplus}$, as obtained from equation (48). With increasing time the curves approach the final period curve, shown as a dotted line. Physically the $G^{\oplus}$ curves represent the distribution of the total thermal "energy" $\overline{T^2}$ among thermal "eddies" of size $\sim 1/\kappa^{\oplus}$.

Since, as a first approximation theory, the theory presented here is restricted to not-too-

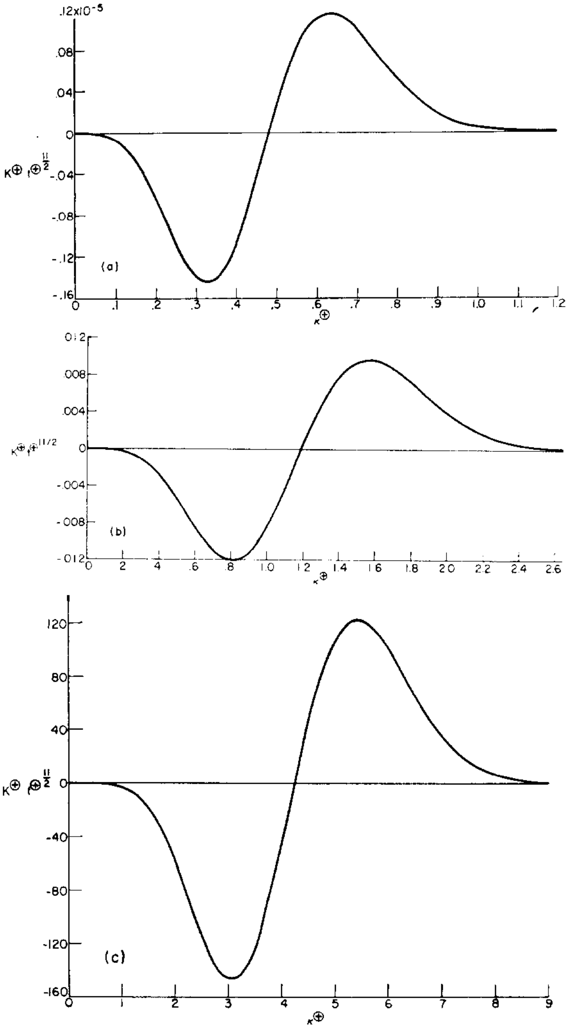


FIG. 5. Variation of $K^{\oplus}t^{\oplus 11/2}$ with wave number. (a) $Pr = 0{\cdot}1$. (b) $Pr = 0{\cdot}72$. (c) $Pr = 10$.

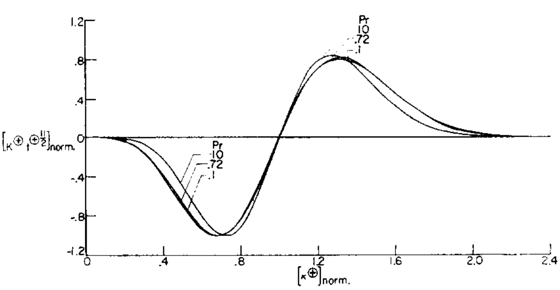


FIG. 6. Same results as in Fig. 5, but with ordinate and abscissa normalized.

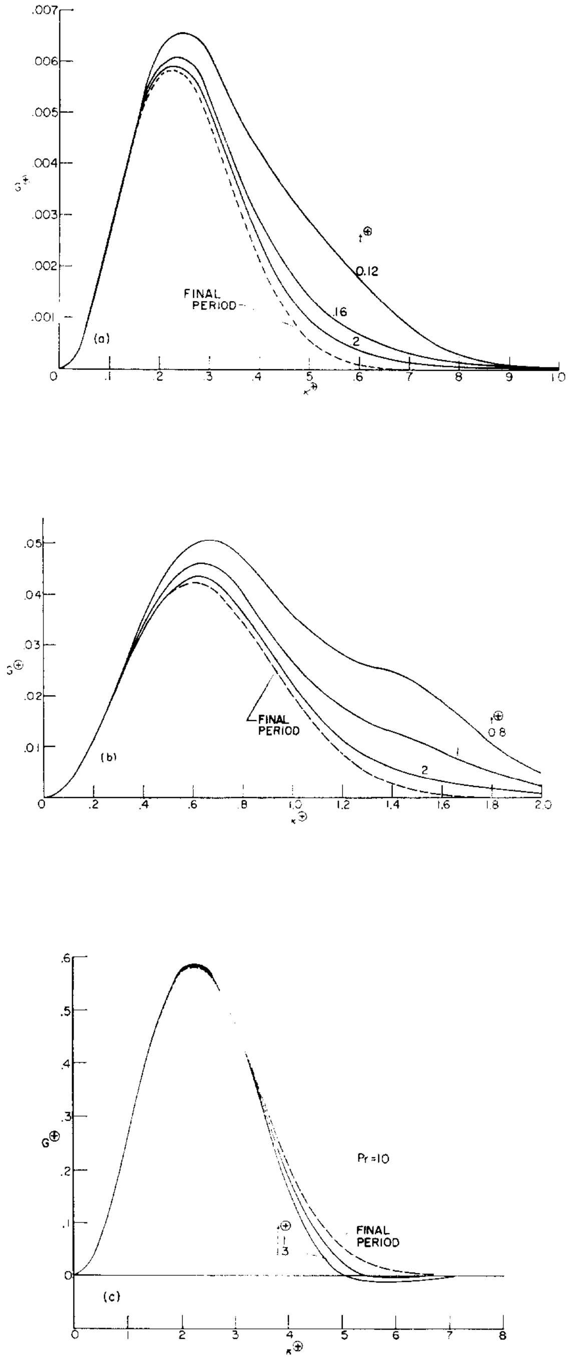


FIG. 7. Dimensionless spectral energy function $G^{\oplus}$ as a function of wave number and time. (a) $Pr = 0{\cdot}1$. (b) $Pr = 0{\cdot}72$. (c) $Pr = 10$.

large convection/conduction ratios and since (for a given Reynolds number) increasing Prandtl number signifies an increasing value of this ratio, it might be expected that the theory would apply less well for higher values of Prandtl number. Figs. 7(a), (b) and (c) indicate that this is apparently the case: the curves for $Pr = 0{\cdot}1$ are quite smooth, those for $Pr = 0{\cdot}72$ and small values of $t^{\oplus}$ are a little less smooth and for $Pr = 10$ the spectral curves begin to go negative at the smaller values of $t^{\oplus}$.

The reason that the final period curve is approached from below for large $\kappa^{\oplus}$ in Fig. 7(c) is that at such high values of Prandtl number the transfer term supplies thermal "energy" to the high $\kappa^{\oplus}$ range faster than it can be dissipated—hence $[\partial G^{\oplus}/\partial t^{\oplus}]_{\kappa \oplus}$ is positive instead of negative as with the lower Prandtl numbers.

If equation (33) is integrated with respect to κ from 0 to ∞ and use is made of equations (35) and (42), the resulting equation is

$$-\frac{\partial(\overline{T^2}/2)}{\partial t} = \frac{2\nu}{Pr}\int_0^\infty \kappa^2 G \, d\kappa$$

This equation points out the interesting fact that for a given viscosity and temperature fluctuation spectrum the decay rate is inversely proportional to the Prandtl number. The results of this analysis, however, are not comparable on this basis since the manner in which the initial conditions were imposed (equation (31)) precludes comparing two different Prandtl number fluids with the same spectral curve. However, the results of this analysis do show that the decay rate decreases relative to the final period rate with increasing Prandtl number, as can be seen from Figs. 8(a), (b), and (c), where equation (49) has been plotted for values of Prandtl number of 0·1, 0·72 and 10. The rates of decay for $Pr = 0{\cdot}1$ and 0·72 are greater than that predicted by the final period law (dotted line) while that for $Pr = 10$ is less than the final period rate. The Prandtl number function R of equation (49) changes from positive to negative at $Pr = 4{\cdot}7717$ for which $R = 0$ and the decay rate is exactly that for the final period. Although the decay rate for $Pr = 4{\cdot}7717$ is described by the final period equation, the spectral curves are not of the final period shape (except for the final period). It just happens that each of the spectral curves for this value of Prandtl number lies partly above and partly below the final period curve in such a way that the integral for $\overline{T^2}/2$

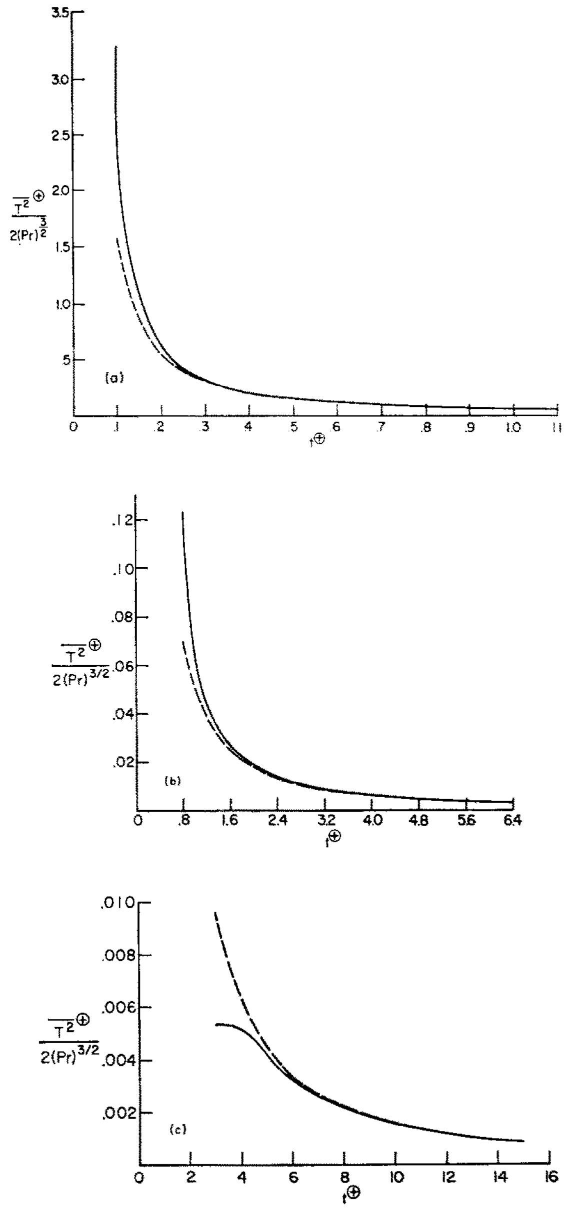


FIG. 8. Dimensionless square of temperature fluctuation as a function of time. ——— Equation (49). — — — — Final period. (a) $Pr = 0{\cdot}1$. (b) $Pr = 0{\cdot}72$. (c) $Pr = 10$.

indicated by equation (42) is equal to the final period value.

Corrsin [1] has previously pointed out that for the final period, as well as for self-preserving and inertial spectrums at very large Reynolds and Peclet numbers, temperature fluctuations die out more slowly than velocity fluctuations. This analysis indicates that the same is true for times before the final period, as can be seen by comparison of equation (43) for $\overline{T^2}$ with the analogous equation for $\overline{u^2}$ (equation (38) of [3]).

CONCLUSIONS

The principal conclusions to be drawn from the present work appear to be the following:

(1) It was possible to represent the available experimental temperature fluctuation decay data by means of the present theory.

(2) For times before as well as during the final period, the temperature fluctuations decay more slowly than do the velocity fluctuations.

(3) The general effect of increasing Prandtl number is to extend the spectral function to larger wave numbers and to reduce the rate of decay of the temperature fluctuations.

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